EOSC - 213
Computational methods in geological engineering
Review of PDEs Quiz

N. Seigneur

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Analogy to heat equation

For mass, the continuity equation writes:

$$\frac{\partial c}{\partial t} + \vec{\nabla} \cdot \vec{j} = Q. \quad (1)$$

Let us consider an energy problem. The continuity equation writes:

$$\frac{\partial x}{\partial t} + \vec{\nabla} \cdot \vec{j} = Q. \quad (2)$$

By analogy, what do you think the units of x should be if this represents an energy conservation problem?

(a) J
(b) J/m^3
(c) J/s
(d) J/m^3/s

Analogy to heat equation

Let us consider an energy problem. x is replaced by E, the energy by unit of volume. The continuity equation writes:

$$\frac{\partial E}{\partial t} + \overrightarrow{\nabla} \cdot \overrightarrow{j} = Q. \quad (3)$$

To which physical variable could you link E?

(a) Pressure p

(b) Temperature T

(c) Velocity v

(d) Electric current i

Analogy to heat equation

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$$\frac{\partial E}{\partial t} + \overrightarrow{\nabla} \cdot \overrightarrow{j} = Q. \quad (3)$$

To which physical variable could you link E?

(a) Pressure p
(b) Temperature T
(c) Velocity v
(d) Electric current i

We can show that we can write:

$$\Delta E = \rho c_p \Delta T, \quad (4)$$

with ρ the density, c_p the heat capacity $J/kg/K$ which represents (a high c_p says that a large amount of energy is required to warm a certain quantity of matter by 1 degree).

Let us consider an energy problem.

$$\frac{\partial \rho c_p T}{\partial t} + \overrightarrow{\nabla} \cdot \overrightarrow{j} = Q. \tag{5}$$

By analogy, what do you think the units of j should be if this represents an energy conservation problem?

(a) J/m^2
(b) J/m^3/s
(c) J/s
(d) J/m^2/s

j is the total heat flux, which arises from temperature gradients, convective heat-fluxes, ...

Analogy to heat equation

Let us consider the energy problem conservation in the classroom:

$$\frac{\partial \rho c_p T}{\partial t} + \overrightarrow{\nabla} \cdot \overrightarrow{j} = Q. \tag{6}$$

By analogy, what do you think the units of Q should be if this represents an energy conservation problem?

(a) J

(b) J/m^3

(c) J/s

(d) J/m^3/s

Analogy to heat equation

Let us consider the energy problem conservation in the classroom:

$$\frac{\partial \rho c_p T}{\partial t} + \overrightarrow{\nabla} \cdot \overrightarrow{j} = Q. \qquad (7)$$

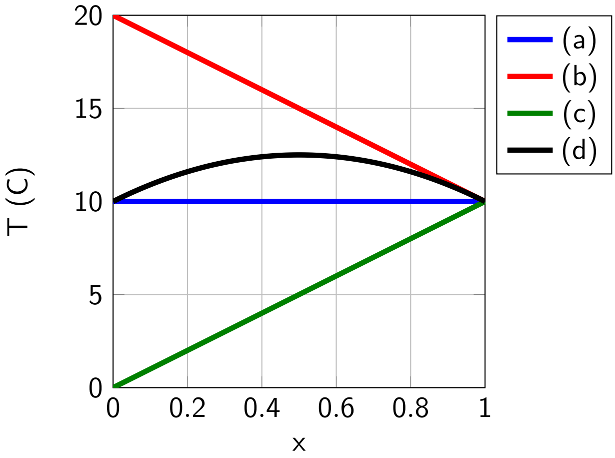
Fourier's law is similar to Fick's law of diffusion. Which of the next law would you guess is Fourier's law?

(a) $\overrightarrow{j} = k\overrightarrow{\nabla} T$
(b) $\overrightarrow{j} = kT$
(c) $\overrightarrow{j} = -k\overrightarrow{\nabla} T$
(d) $\overrightarrow{j} = -k\frac{\partial^2 T}{\partial x^2}$

$$\frac{\partial \rho c_p T}{\partial t} = \overrightarrow{\nabla} \cdot k\overrightarrow{\nabla} T + Q. \qquad (8)$$

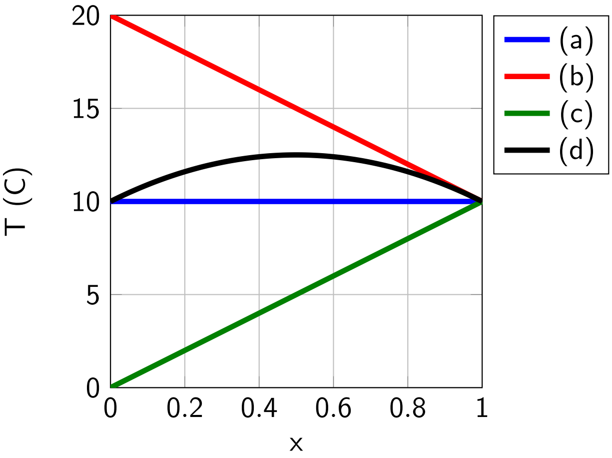
Predicting the solution

If we consider that all the walls are **impermeable** to heat, that the temperature outside the window (on the right) is constant in time at $T = 10$, what would be the final temperature profile in the room?



Predicting the solution

If you all contribute to some heat generation, that the classroom is located in an oustide environment maintainted at 10C, which is the corresponding profile?



Predicting the solution

Consider a closed system with 1 liter of water at a temperature of 20C. What happens if you add 1 liter of water at 40C ?

(a) Final temperature is 20C
(b) Final temperature is 30C
(c) Final temperature is 40C
(d) Final temperature is 50C

Consider a closed system with 1 liter of water at a temperature of 20C. What happens if you add 1 liter of water at 40C ?

(a) Final temperature is 20C
(b) Final temperature is 30C
(c) Final temperature is 40C
(d) Final temperature is 50C

Consider a very large system of water at temperature of 20C. What happens if you add 1 liter of water at 80C?

(a) Final temperature is 20C
(b) Final temperature is 30C
(c) Final temperature is 40C
(d) Final temperature is 50C

In the 2D transient notebook assignment, we basically solved the equation

$$\frac{\partial c}{\partial t} = \operatorname{div}\left(D\,\overrightarrow{\operatorname{grad}} c\right) + Q_m. \tag{9}$$

The heat equation is the same:

$$\frac{\partial \rho c_p T}{\partial t} = \operatorname{div}\left(k\,\overrightarrow{\operatorname{grad}} T\right) + Q_h. \tag{10}$$

and Darcy-equation is the same

$$S_s\frac{\partial h}{\partial t} = \operatorname{div}\left(K\,\overrightarrow{\operatorname{grad}} h\right) + Q_w, \tag{11}$$

where K is the hydraulic conductivity, S_s is the storage coefficient, Q_w represents water sources, and $S_s\frac{\partial h}{\partial t}$ represents the variation in the volumetric water content.

Homework

You can use all the routines in the 2D transient notebook (you can also work in 1D) to solve these problems, with non-uniform source-terms, with non-uniform flux-coefficient (diffusion D, heat-conductivity k, hydraulic conductivity K). Several boundary conditions can be applied:

1. Dirichlet: specified concentration c, temperature T, hydraulic head h (which can vary in time)
2. Neumann: specified flux (of mass, heat, water), which corresponds to the derivative of the quantity (a zero flux depicts an impermeable layer).

Homework

We strongly encourage you to play with these functions on these different topics to build your physical intuition. You can start by any initial condition you want, any boundary conditions you like, but you should be able to predict how the system should evolve.

A couple of examples:

- Heat problem: predict the final temperature profile of a system with an initial non-uniform temperature
- Heat problem: show how the temperature profiles evolve for different source terms or conductivity value
- Darcy: compute pressure profiles