EOSC 213 Quiz 1 February 5^{th}, 2019
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EOSC 213 - Quiz

Instructions (20 points in total)

- Read the examination before beginning.
- Calculators are allowed (if you don't have one, just give the expression to type in a calculator).
- You have exactly 30 minutes for the examination.
- Be as precise and clear as possible.
- This is a closed book examination.
- If you get stuck, make an assumption, state what it is and try to carry on.

Let us consider the following ODE, where t represents time (expressed in seconds):

$$\frac{dy}{dt} + 2y = 1 \tag{1}$$

and the following functions

$$\begin{array}{rcl} y_1(t) & = & A\exp(-2t) \\ y_2(t) & = & \frac{1}{2} \\ y_3(t) & = & y_1(t) + y_2(t) \end{array} \tag{2}$$

Q1 Describe the ODE 1 (linearity, order, number of solutions, asymptotic behaviour?) [**4 points**]

This is a first order ODE because the highest derivative of y is its first derivative. This is a linear equation because y and its derivatives are never affected to a power different than 1. It has an infinite amount of solutions. The asymptotic solution is 1/2

Q2 Prove that the function y_1 is a solution to the homogeneous ODE associated to the ODE 1.[**2 points**]

The derivative of $y_1(t)$ is

$$\frac{dy_1}{dt} = \frac{d}{dt}\left(A\exp(-2t)\right) = -2\overbrace{A\exp(-2t)}^{y_1} = -2y_1$$

So $\frac{dy_1}{dt} + 2y_1 = -2y_1 + 2y_1 = 0$ which solves the homogeneous problem.

Q3 Prove that the function y_2 is a particular solution to the ODE 1.[**2 points**]

Since $\frac{dy_2}{dt} = 0$, $2y_2 = 2 \times \frac{1}{2} = 1$ so y_2 is a solution to the ODE.

Q4 Prove that the function y_3 is a solution to the ODE 1.

The derivative of y_3 is:

$$\frac{dy_3}{dt} = \frac{d}{dt}\left(A\exp(-2t) + \frac{1}{2}\right) = -2A\exp(-2t)$$

Then, we have:

$$\frac{dy_3}{dt} + 2y_3 = -2A\exp(-2t) + 2\left(A\exp(-2t) + \frac{1}{2}\right) = 1$$

Consider the following differential problem.

$$\begin{cases} \dfrac{dy}{dt} + 2y &= 1 \\ y(0) &= 1 \end{cases} \tag{3}$$

Q5 What should be the value of A (in $y_1(t)$) so that $y_3(t)$ satisfies the differential problem 3? [**1 point**]

$A = 1/2$ so that $y_3(0) = 1/2 + 1/2\exp(-0) = 1$

Q6 Using forward Euler (with a timestep of $\Delta t = 0.1$ s), compute the value of $y(t)$ which solves the differential problem 3:

(i) after 0.1 second ; [**1 point**]

(ii) after 0.2 second. [**1 point**]

Foward Euler's method yields:

$$\begin{array}{lcllcl} y(\Delta t) &=& y(0) + \Delta t(1 - 2 * y(0)) &\Rightarrow& y(0.1) &=& 0.9 \\ y(2\Delta t) &=& y(\Delta t) + \Delta t(1 - 2 * y(\Delta t)) &\Rightarrow& y(0.2) &=& 0.82 \end{array}$$

(iii) Give an estimate of the relative error (in %) after 0.1 second. [**1 point**]

(iv) Give an estimate of the relative error (in %) after 0.2 second.[**1 point**]

The real value of y_{real} after 0.1 second is $\frac{1}{2}(1 + \exp(-2 \times 0.1) \approx 0.909$, and the real solution after 0.2 second is ≈ 0.835. The relative error is:

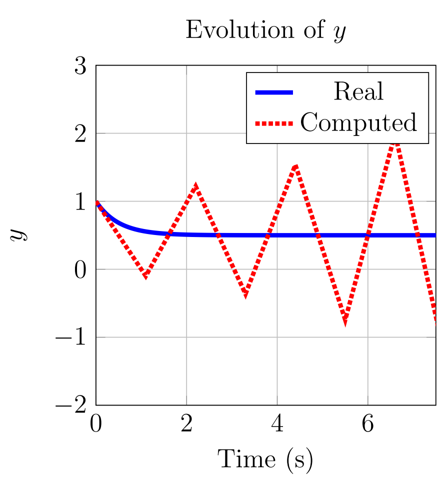
$$\epsilon(\%) = 100\% \frac{|y - y_{\text{real}}|}{y_{\text{real}}}$$

so the errors are respectively 1 and 1.8 %.

Q7 Another student wanted to solve the evolution of y for 10 seconds, using a constant timestep. Knowing you're an expert in Euler's method, he gives you this code and complains it is not working. Find his mistakes and feel free to advise him on his programming skills. [**3 points**]

```
dt = 0.1 # timestep in seconds
Tf = 10 # simulation time in seconds
n = int(1+Tf/dt) # number of timesteps
y = np.zeros(n,float)
y[0] = 1
for i in range(n-1):
    y[i+1] = y[i] + dt * ( 1 - 2 * y[i])
```

Q8 After you have successfully helped him, he comes back later and says you were probably wrong because his solution looks weird. What advice are you going to give him? [**2 points**]



These are the typical instabilities observed when using forward Euler's methods with a too large timestep! Decrease the timestep!