EOSC 213 Quiz 4 March 28th, 2019
Name: ID:

EOSC 213 - Quiz

Instructions (30 points in total)

- Read the examination before beginning.
- Calculators are allowed (if you don't have one, just give the expression to type in a calculator).
- You have exactly 30 minutes for the examination.
- Be as precise and clear as possible.
- This is a closed book examination.
- If you get stuck, make an assumption, state what it is and try to carry on.

Question 0: English is a funny language test

Q0 Complete the sentence with your favourite word. *If vegetarians consume vegetables, then humanitarians consume .....* [**1 point**]

Question 1: Finite-difference approximations

Let us consider the exponential function described in equation 1

$$f(x) = \exp(x). \quad (1)$$

Q1a Compute a first-order approximation to the first derivative of the exponential function at $x = 0$ ($f'(0)$) using $\Delta x = 0.2$. [**2 points**]

$\exp(0.2) = 1.22 \rightarrow f'(0) \approx \frac{\exp(0.2)-\exp(0)}{0.2} = 0.22/0.2 = 1.1$. The exact value is 1, so we have a 10% relative error.

Q1b Compute a second-order approximation to the first derivative of the exponential function at $x = 0$ ($f'(0)$) using $\Delta x = 0.2$. [**2 points**]

$\exp(0.2) = 1.22$ and exp(-0.2) = 0.819. $\rightarrow f'(0) \approx \frac{\exp(0.2)-\exp(-0.2)}{0.4} = 1.007$. The exact value of the derivative is equal to 1, so we have less than 1% error.

Question 2: Diffusion Boundary Value Problem

The 1D transient diffusion equation with homogeneous diffusion coefficient can be written as:

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}. \quad (2)$$

Let us consider the physical domain $x \in [-1; 1]$ m, with the specified (Dirichlet) boundary conditions $c(1, t) = c(-1, t) = 0$ (where, for example, $c(1, t)$ should be read as the concentration at $x = 1$ and all times t) and initial condition $c(x, 0) = c_0 \cos\left(\frac{\pi x}{2}\right)$. Consider the following function:

$$c(x, t) = c_0 \cos\left(\frac{\pi x}{2}\right) \exp(-\alpha t) \quad (3)$$

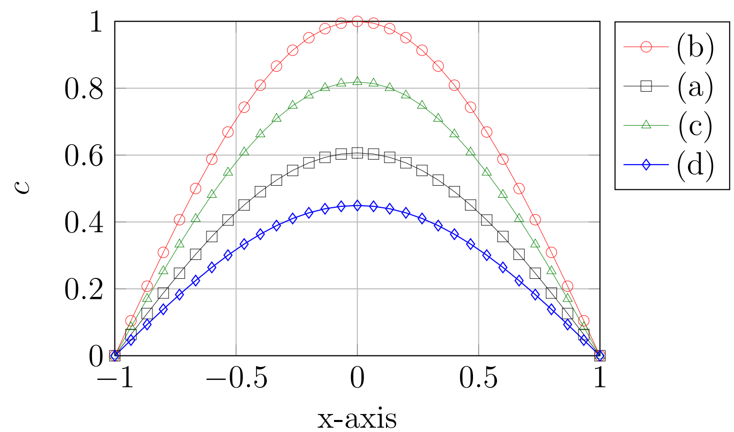
Q2a Show that the function described in equation 3 satisfies the two boundary conditions at all times **[2 points]**

$\cos(\frac{\pi}{2}) = \cos\left(\frac{-\pi}{2}\right) = 0$. So, no matter the value of t, $c(1,t) = c(-1,t) = 0$

Q2b Show that the function described in equation 3 satisfies the partial differential equation 2 at every point in the domain. **Justify** your answer. **[2 points]**

$\frac{\partial^2 c(x,t)}{\partial x^2} = -\frac{\pi^2}{4}c(x,t)$ and $\frac{\partial c(x,t)}{\partial t} = -\alpha c(x,t)$. So, if we plug that in the equation, we have: $\alpha = D\frac{\pi^2}{4}$. So function 3 solves the problem for every point, provided the relation $\alpha = D\frac{\pi^2}{4}$ is satisfied.

Q2c Concentrations were measured at four different times and are represented in the graph below. The messy person who did the measurements does not remember at which time these measurements were taken. Can you help him, using your physical intuition? Label the 4 curves in termporal order where $t_1 < t_2 < t_3 < t_4$. **[2 points]** (b) (c) (a) (d)



Q2d What can you say about the change in the total mass in the system with time? Is that consistent with the boundary conditions? Explain. **[2 points]** The total mass is decreasing (the area under the concentration curves decreases! That is consistent because boundary conditions state that concentration is 0 at -1 and 1, so that does not prevent mass from going out of the system. (In the previous quiz, zero-flux boundary conditions were applied: if you put a sugar in your cup of coffee / tea, it will spread but not go out of your cup. But if you put the sugar in the ocean, and you focus only on small fraction of it, at late times, you won't notice that you put sugar in there in the first place!)

Q2e Use your physical reasoning (and equation 3) to describe the asymptotic/final solution (concentration versus x).. **[2 points]**

1D, homogeneous diffusion, boundary conditions are constant and 0: the asymptotic solution will be linear. Considering the boundary conditions, it will be horizontal, and at zero (all mass will be out of the system).

Question 3: Conservation equation The general conservation (or sometimes called continuity) equation says:

$$\frac{\partial\,\text{stuff}}{\partial t} = -\vec{\nabla}\cdot\vec{j}, \tag{4}$$

or, in cartesian coordinates

$$\frac{\partial\,\text{stuff}}{\partial t} = -\frac{\partial j_x}{\partial x} - \frac{\partial j_y}{\partial y} - \frac{\partial j_z}{\partial z}. \tag{5}$$

$\overrightarrow{j}$ is a flux vector that describes the rate at which "stuff", the conserved quantity, fluxes (moves).

Q3a Provide two examples of what "stuff", the **conserved quantity**, could represent. [**2 points**]

Mass, energy (but also electric charge, number of particles, ...). Not concentration!

Q3b Write the PDE conservation law (either in nabla notation or in cartesian coordinates) for the case of the diffusion of a solute (not in a porous media) where the flux is given by Fick's law $\overrightarrow{j_{\text{diff}}} = -D\overrightarrow{\nabla}c$ [**3 points**]

$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}D\frac{\partial c}{\partial x} + \frac{\partial}{\partial y}D\frac{\partial c}{\partial y} + \frac{\partial}{\partial z}D\frac{\partial c}{\partial z}$

Q3c What is the physical meaning of the term $\frac{\partial \text{stuff}}{\partial t}$ in the case of a diffusing solute? Provide a one or two sentence(s) explanation. [**2 points**]

It represents the change in mass per unit of volume. It represents how the mass storage changes with time, from the different in and out-fluxes or sources/sinks terms.

Q3d Write the PDE conservation law (either in nabla notation or in cartesian coordinates) for the case where the flux is given by advection $\overrightarrow{j_{\text{adv}}} = \overrightarrow{v}c$ [**2 points**]

$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}(v_x c) + \frac{\partial}{\partial y}(v_y c) + \frac{\partial}{\partial z}(v_z c)$ or $\frac{\partial c}{\partial t} = \overrightarrow{\nabla} \cdot (\overrightarrow{v}c)$

Question 4: Python

Q4a 3 pts Consider the following dataframe **df**, with rows, columns and index given by:

	temperature	veloc	mass	mass_tenths
date				
2019-03-25 05:00:00+00:00	267.439863	74.146461	0.206719	0.2
2019-03-25 05:10:00+00:00	281.372218	-2.317762	0.611744	0.6
2019-03-25 05:20:00+00:00	278.318157	3.683598	0.296801	0.3
2019-03-25 05:30:00+00:00	266.754425	-83.851746	0.738440	0.7
2019-03-25 05:40:00+00:00	271.826184	-68.338026	0.879937	0.9

In the space below write python statements (single lines) that would return the following:

1. The row corresponding to March 25, 2019 at 5:30 am UCT
2. The temperature for March 25, 2019 at 5:10 am UCT
3. All temperatures in the dataframe

Q4b 3pts Write a python snippet that would use groupby to find the median velocity for all objects with `mass_tenths`=0.7 kg for this dataframe [**3 points**]