

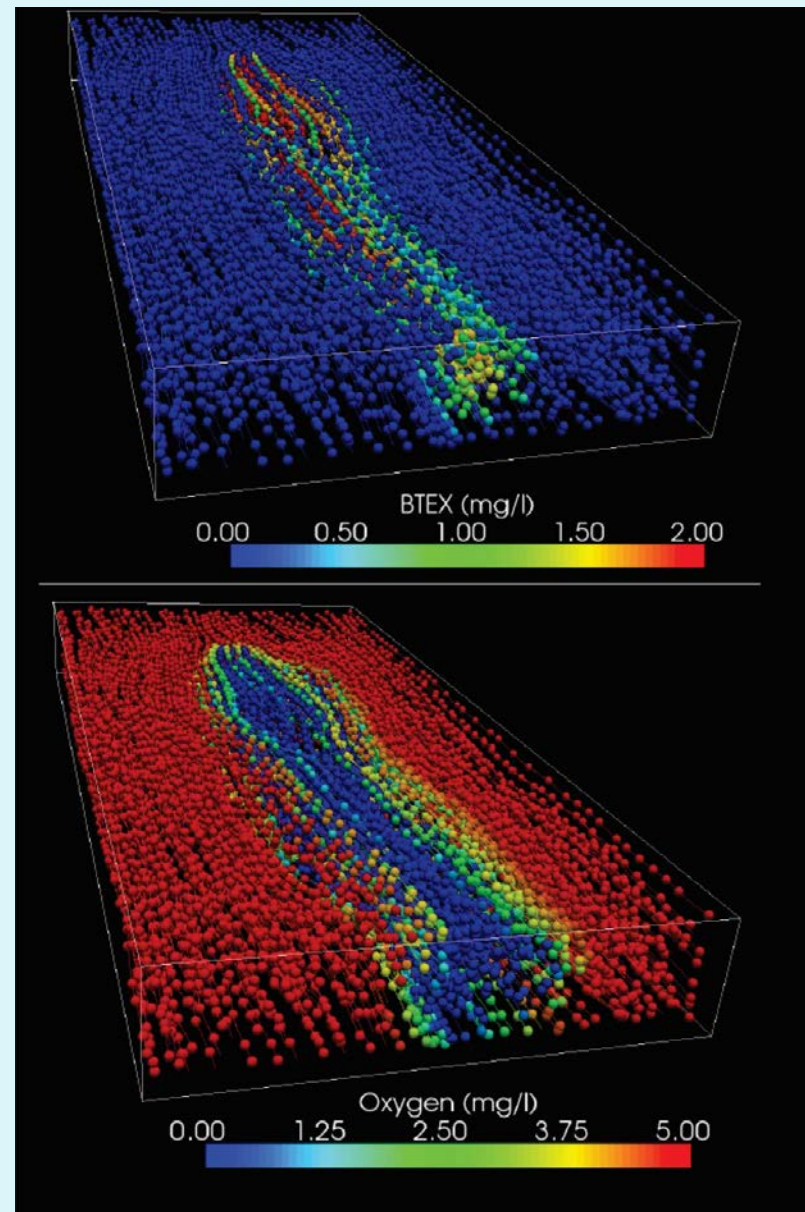
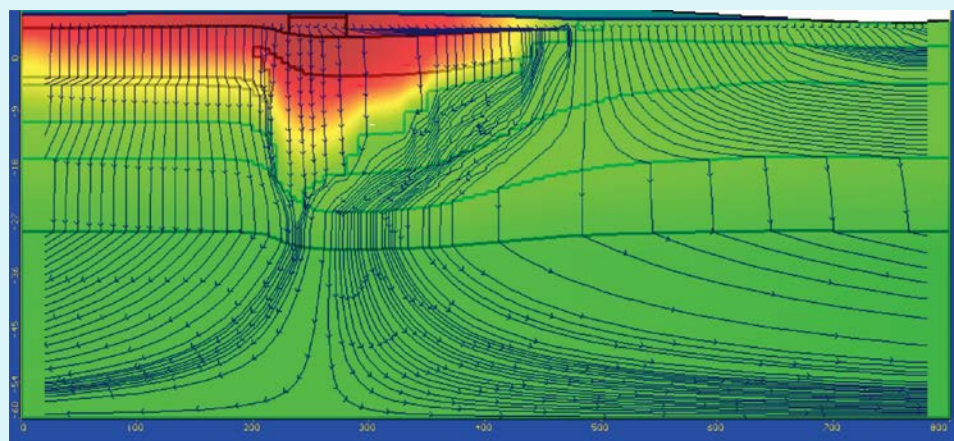
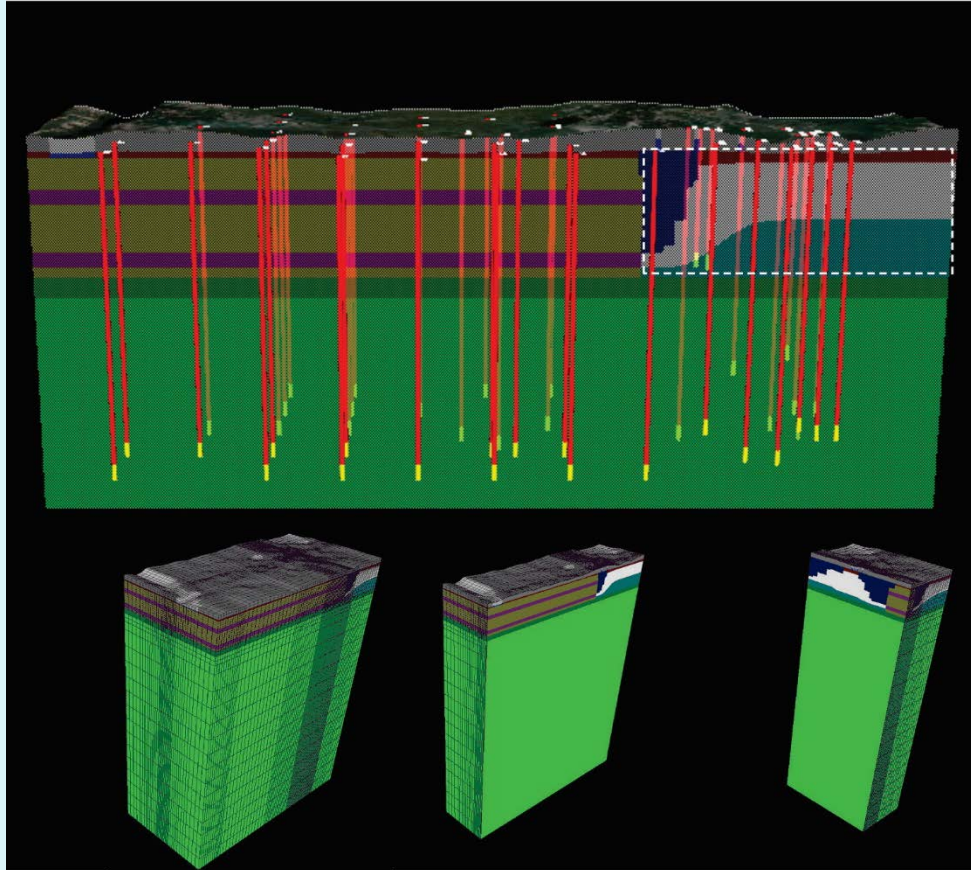
# *EOSC 213*

## *Computational methods in geological engineering*

Combining basic physical principles with their mathematical description and generating computational models that describe them. Introduction to basic building blocks in modeling, simulation and parameter estimation.

**Instructors:** Phil Austin, Roger Beckie, Nicolas Seigneur

**Teaching assistants:** Brian Irwin, Cole Lord-May



# Learning goals

Be able to:

- develop quantitative descriptions of physical systems of interest to geological engineering,
- create or select appropriate algorithms and computational methods to compute system behavior.
- implement computational methods in a programming language (python).

# EOSC 213

<b>Assessment</b>	<b>Assignments/project: 25% Quizzes: 40 %; Participation: 5 %; Final exam: 30%. (Subject to adjustment, depending upon course evolution)</b>
<b>Missed exam and late policy</b>	<p>If you do not write a quiz, for any reason, the remaining evaluations will be summed and renormalized to 100%. <b>NO MAKE-UP QUIZZES WILL BE OFFERED FOR ANY REASON.</b></p> <p>We will consider accepting late deliverables only if a member of the teaching team is notified in advance. Generally, deliverables more than two days late will only be accepted for medical/emergency reasons.</p>
<b>Quizzes</b>	Formats and dates TBD



# How do people learn?

How can a course be designed to maximize student learning?

# How people learn

- **Engagement, participation in:** discussion, speaking, writing, listening, thinking, drawing and doing.
- **Active processing** of material versus passive observation
- Motivation by relevant real-world problems, followed by techniques



<https://www.youtube.com/user/1veritasium>

# Jupyter notebooks

an open-source web application to create and share codes and documents

provides an environment, where you can document your code, run it, look at the outcome, visualize data and see the results without leaving the environment.



# Cocalc

cloud-based workspace

Advantage: uniform software experience

**IMPORTANT ACTION ITEM**

Complete canvas poll





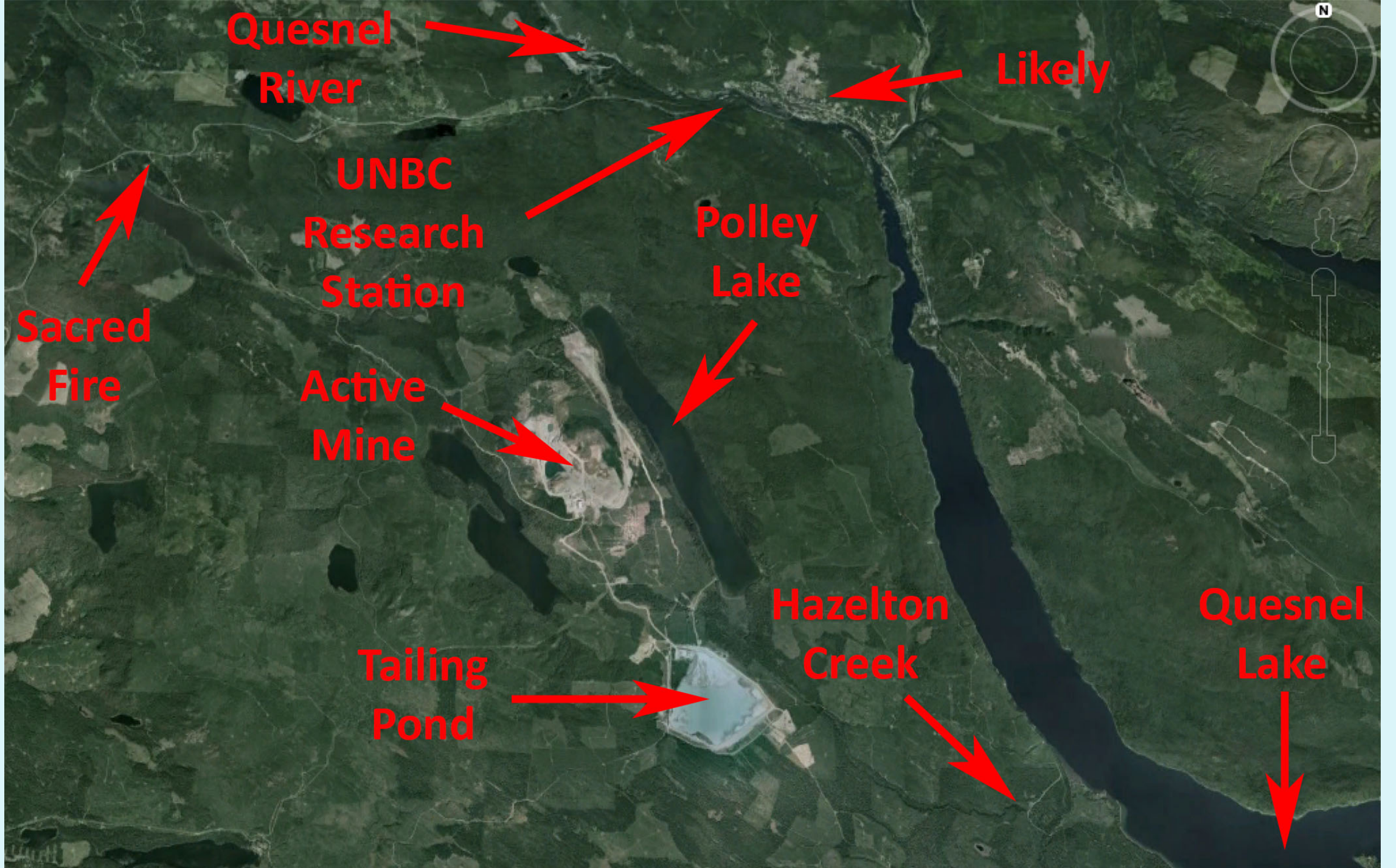
# Introduction to modeling and simulation

# Mount Polley dam failure



# Mount Polley Mine







Mount Polley  
Mine

Polley  
Lake

retention  
basin

Hazeltine  
Creek

Quesnel  
Lake

1 km

**FIGURE 5.1.2: VIEW LOOKING DOWNSTREAM SHOWING UPSTREAM SIDE OF DAM AND REMAINING TAILINGS**



The breach occurred early on August 4, 2014 at a core elevation of El. 969.1 m. Loss of containment was sudden, with no identified precursors.



Mount Polley  
Mine

Polley  
Lake

Hazeltine  
Creek

Quesnel  
Lake

1 km

# Mount Polley mine engineers face disciplinary hearing

Todd Martin - design engineer for the tailings-storage facility, is accused of adopting an overly steep design slope for the perimeter embankment

Laura Fidel - accused of demonstrating unprofessional conduct by accepting the role of engineer of record when she was not qualified by training or experience

Stephen Rice - the most-senior engineer at AMEC Foster Wheeler, is accused of demonstrating unprofessional conduct by allowing Ms. Fidel to act as the engineer of record



Aerial view of the debris going into Quesnel Lake caused by a tailings pond breach near the town of Likely, B.C. Aug. 5, 2014.

JONATHAN HAYWARD/CP

5 Premarket: Apple bon global shares; 'flash cr currencies



# Legal settlement

Knight Piesold

AMEC Foster Wheeler

➔ \$108 M payment to Imperial Metals

# Problem

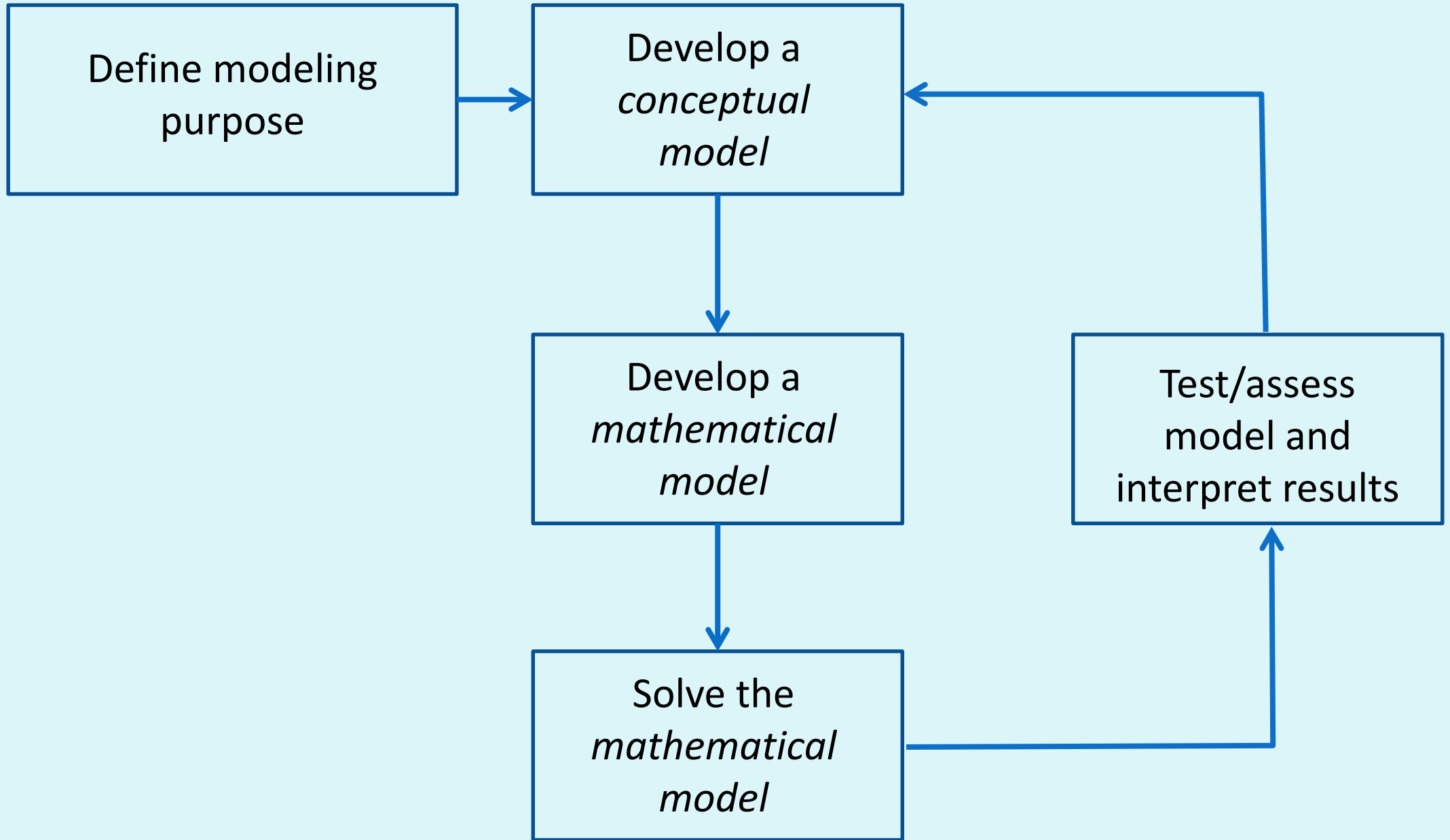
Water balance in a tailings management facility (TMF).

→ Develop a model for the volume of water in the pond with time.

# Problem

- How does the **mass** of water in the pond change with time?

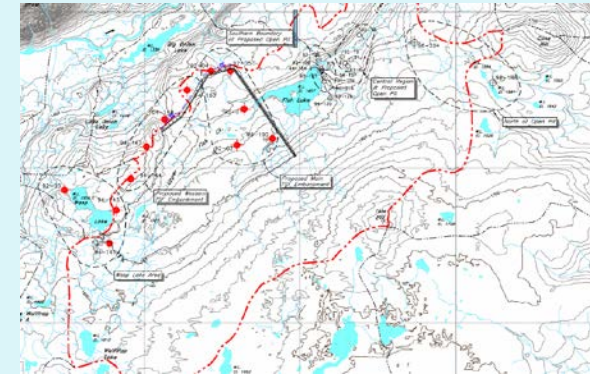
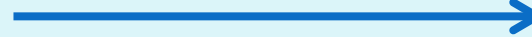




Real world system



Measure

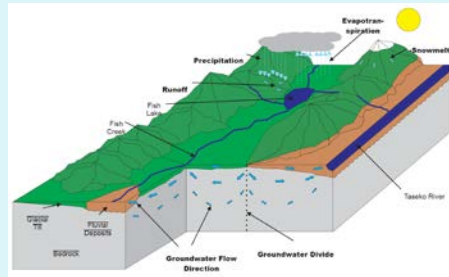


Data

Abstraction (simplify)



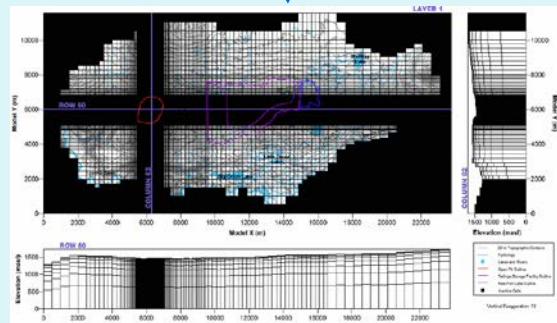
Conceptual model



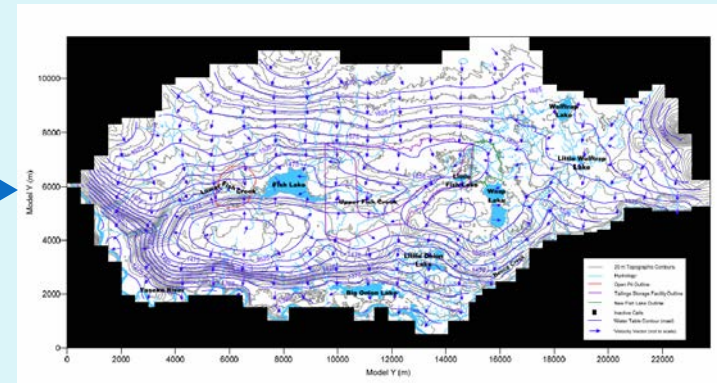
Implement



Computational model



Analysis Simulation



Prediction

Model assessment





# Model

- What makes for a good model?

# Fit for purpose

**George E. P. Box:** “All models are wrong but some are useful.”

**Aristotle:** “It is the mark of an educated mind to rest satisfied with the degree of precision which the nature of the subject admits and not to seek exactness where only an approximation is possible.”

➔ Model should be **fit for its purpose**

# Models and simulation

## Model

- simpler prototype system
- can reproduce some, but not all characteristics of the real-world system
- used to provide insights and predictions about the real system

**Simulate:** use a model to understand or predict system behavior

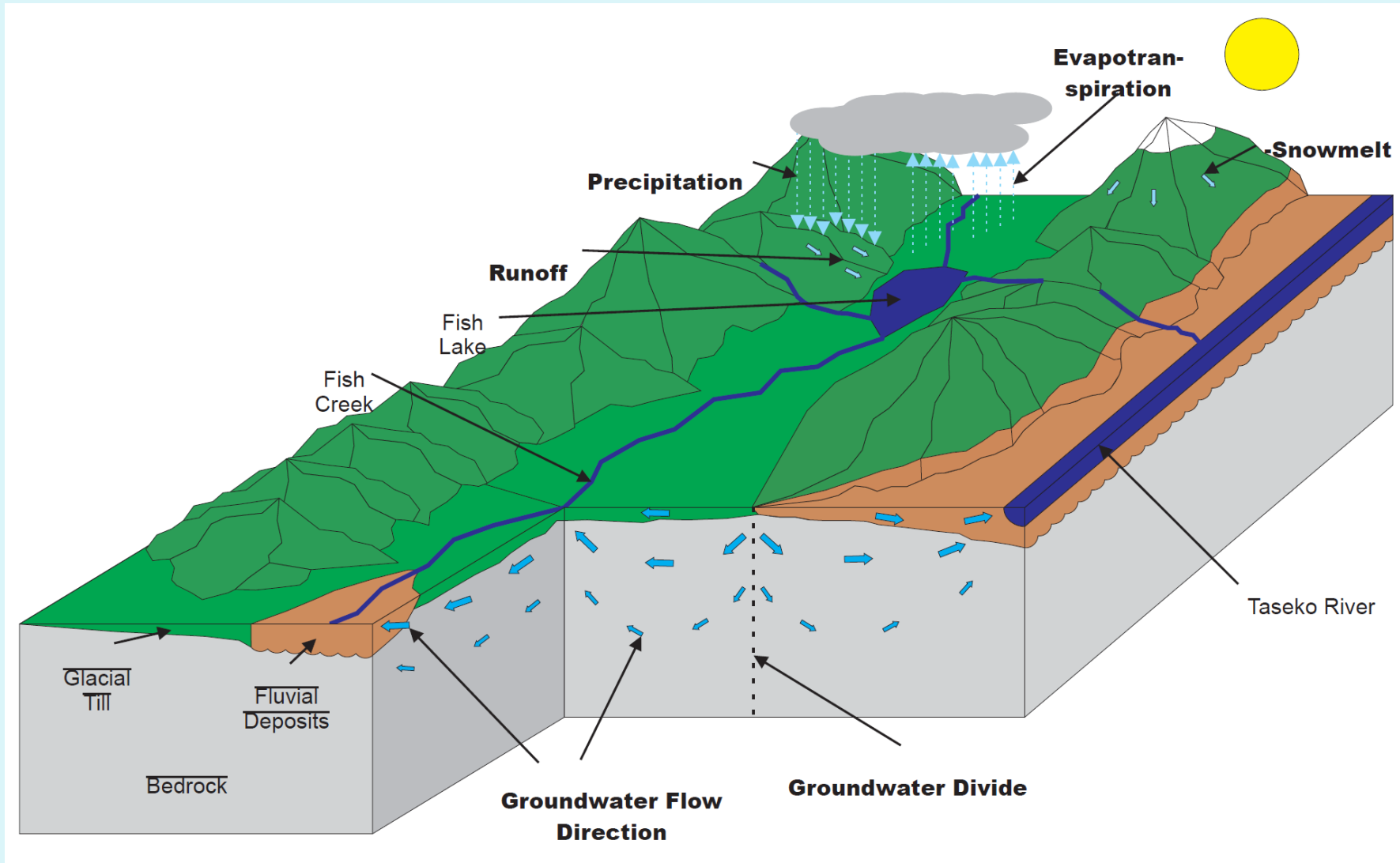


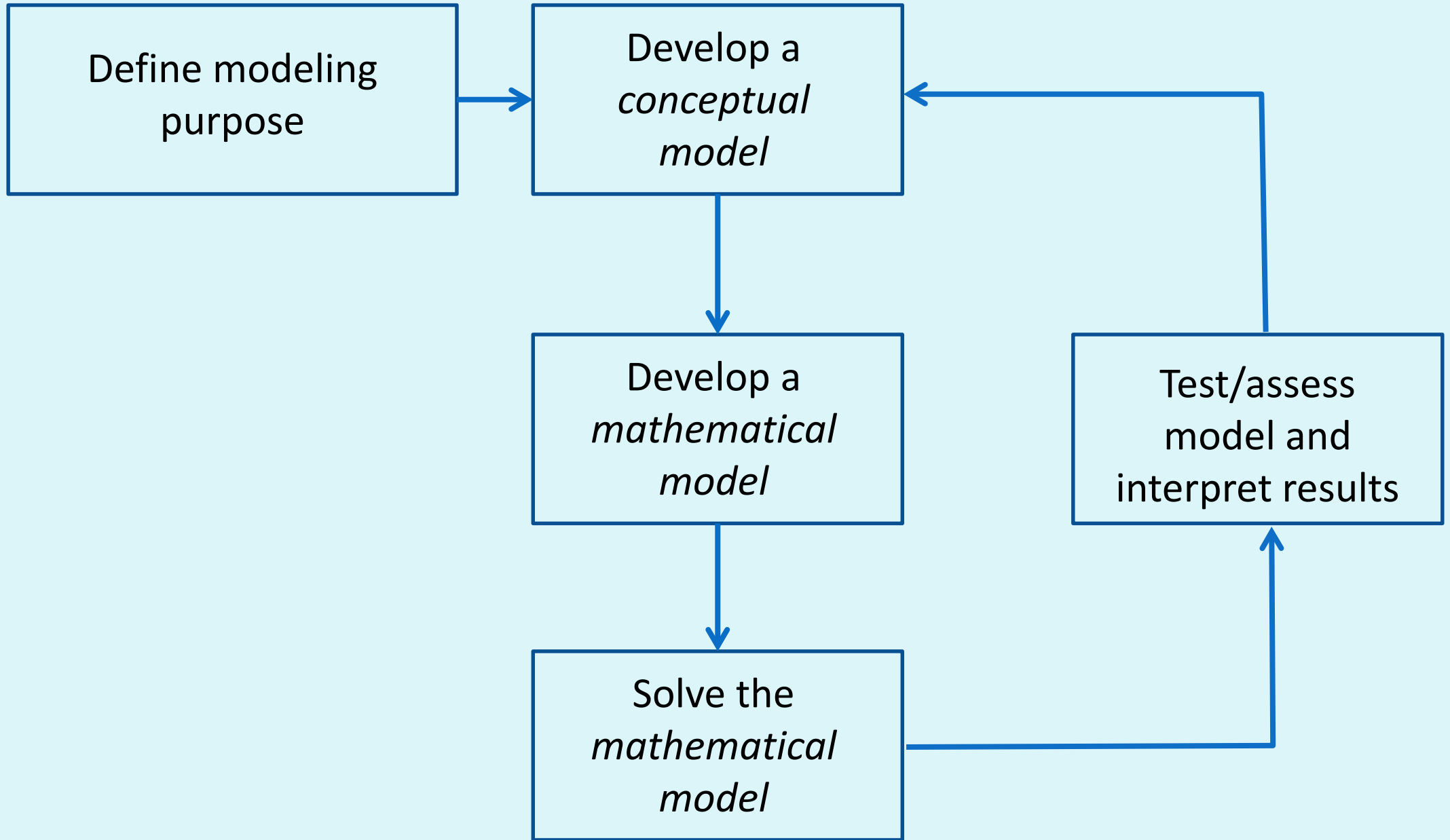
# Conceptual model

A conceptual model of a system specifies the **cause and effect relationships** in the system, and **information on the data required and available** to implement a model.

The conceptual model often includes a **concept map** showing the cause and effect relationships associated with the model and tables showing the different variables, data sources, and references.

# Conceptual model





# Real world





# Conceptual model

A **conceptual model** of a system specifies the **cause and effect relationships** in the system, and **information on the data required and available** to implement a model.

Here: volume of water through time in tailings management facility.



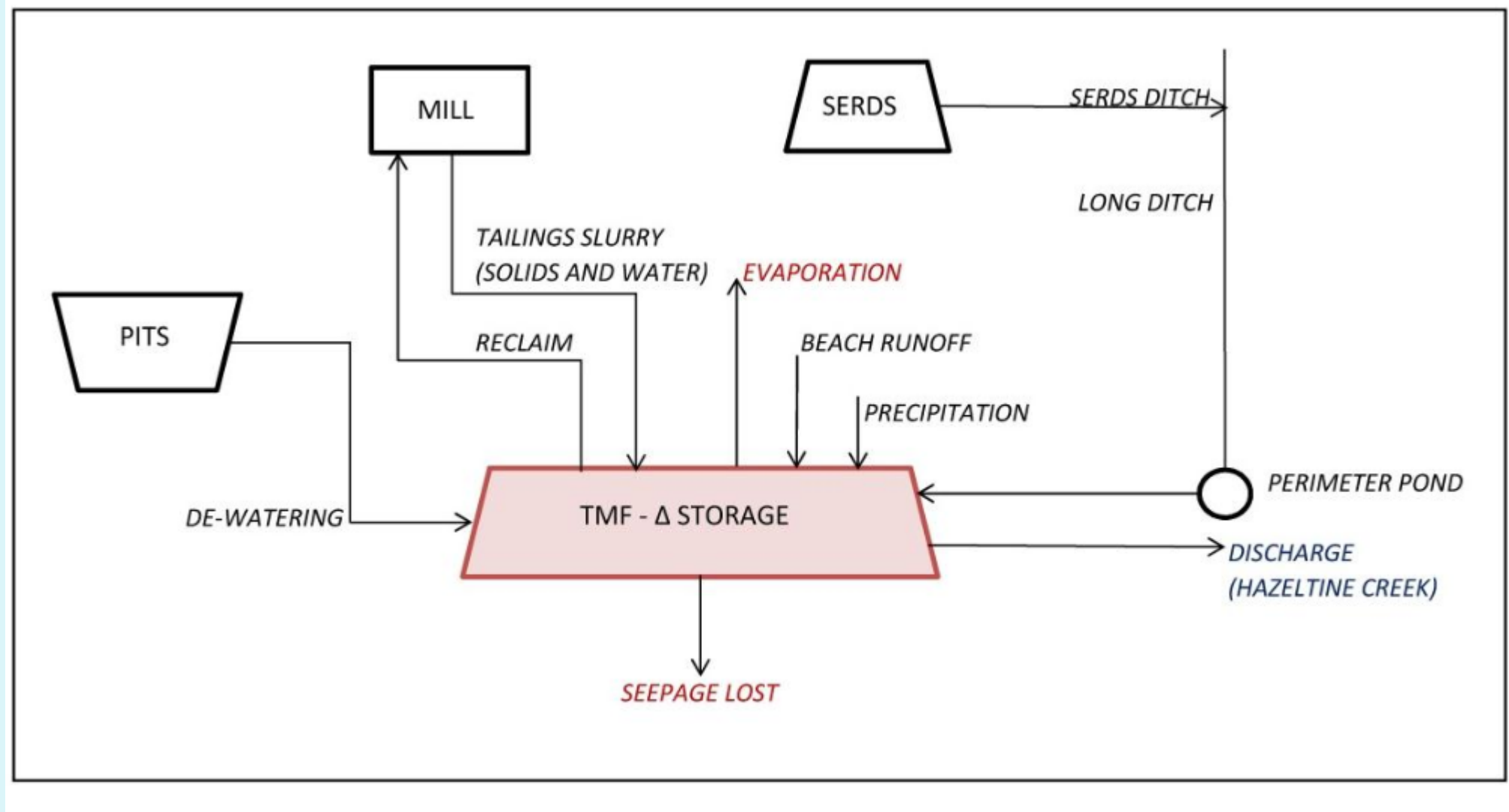
# Conceptual model development



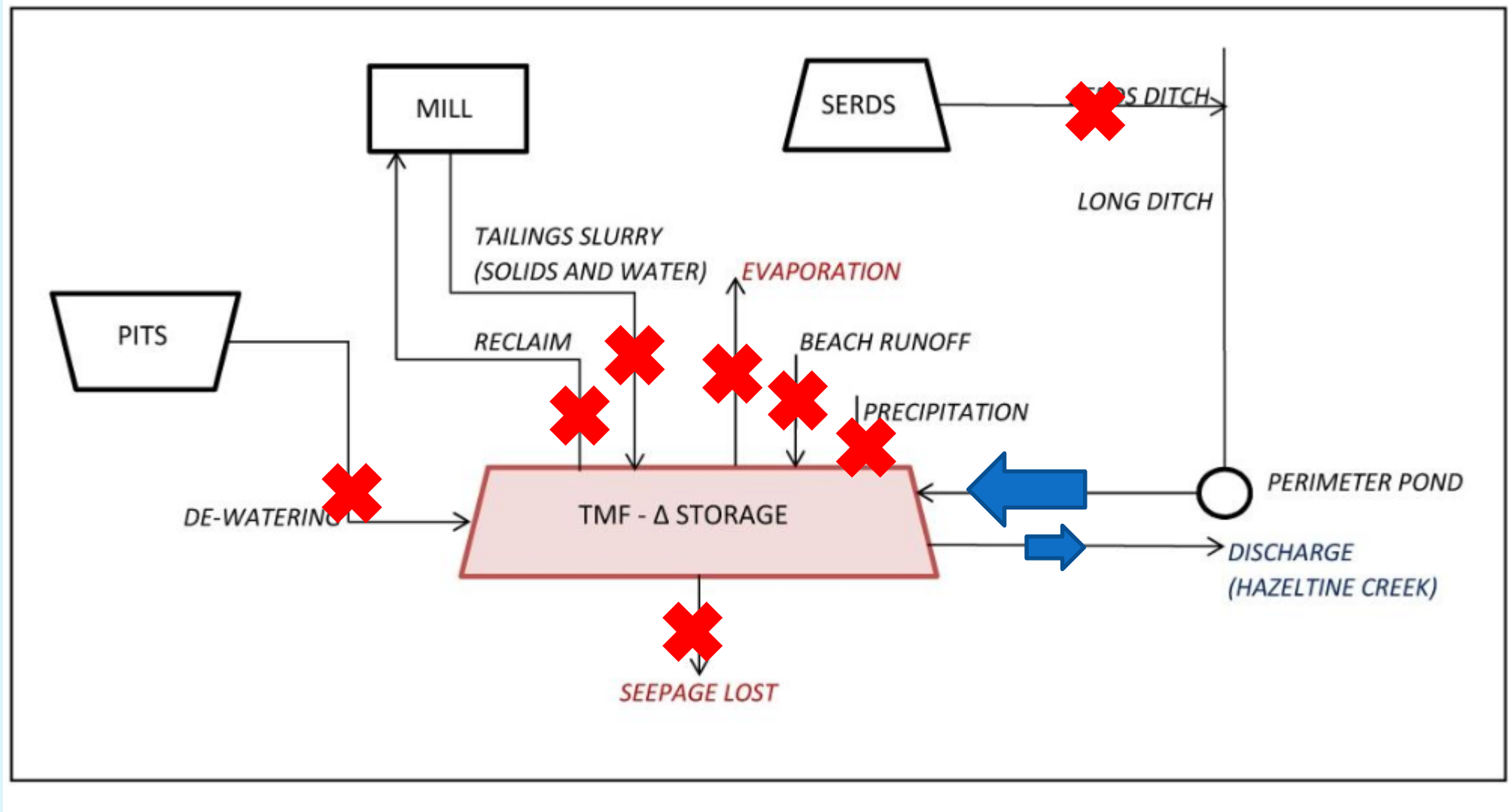
**Inflows and outflows**



# Conceptual water balance



# Simplify: Ignore smaller flows







# Compute *change* in water stored

Compute the change in *mass* of water in TMF **over month of July**

Discharge permit:  $3836 \text{ m}^3 / \text{d}$

Density of water:  $\rho_{\text{water}} = 1000 \text{ kg}/\text{m}^3$

**Table 8: Monthly Flows for Hazeltine Creek**

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Average Flow ( $\text{m}^3/\text{s}$ )	0.05	0.05	0.07	0.74	0.65	0.20	0.10	0.03	0.02	0.02	0.08	0.05
% Flow Distribution	2%	2%	3%	35%	32%	10%	5%	2%	1%	1%	4%	2%



# Mathematical model

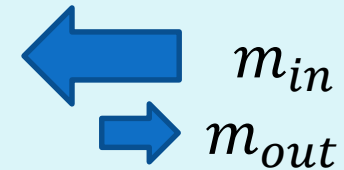
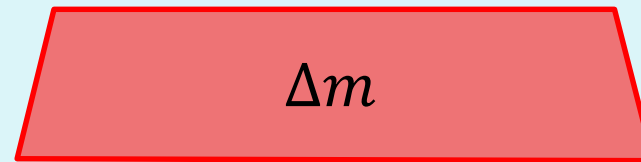
Change in mass in TMF over time  $\Delta t$

$$= (\text{mass in over time } \Delta t) - (\text{mass out over time } \Delta t)$$

Over time period  $\Delta t$ :

$$\Delta m = m_{in} - m_{out}$$

Conceptual model





# Solution

## Mass in calculation

Volumetric flow rate **in** during July:

Total volume **in** during July (31 days):

Total *mass in* during July:

## Mass out calculation

Volumetric flow rate **out** during July:

Total volume **out** during July (31 days):

Total *mass out* during July:



Why use mass and not volume?



# Model for volume of water in TMF

Over time  $\Delta t$ :

$$\Delta m = m_{in} - m_{out}$$

With  $\rho \left[ \frac{M}{L^3} \right]$  density of water;  $V [L^3]$  volume of water in TMF,  $Q \left[ \frac{L^3}{T} \right]$  volumetric flow rate

Then  $\Delta m = \rho \Delta V$ ;  $m_{in} = \rho Q_{in} \Delta t$ ;  $m_{out} = \rho Q_{out} \Delta t$

So

$$\rho \Delta V = \rho Q_{in} \Delta t - \rho Q_{out} \Delta t$$

If **density is constant** (water assumed incompressible), then can simplify to

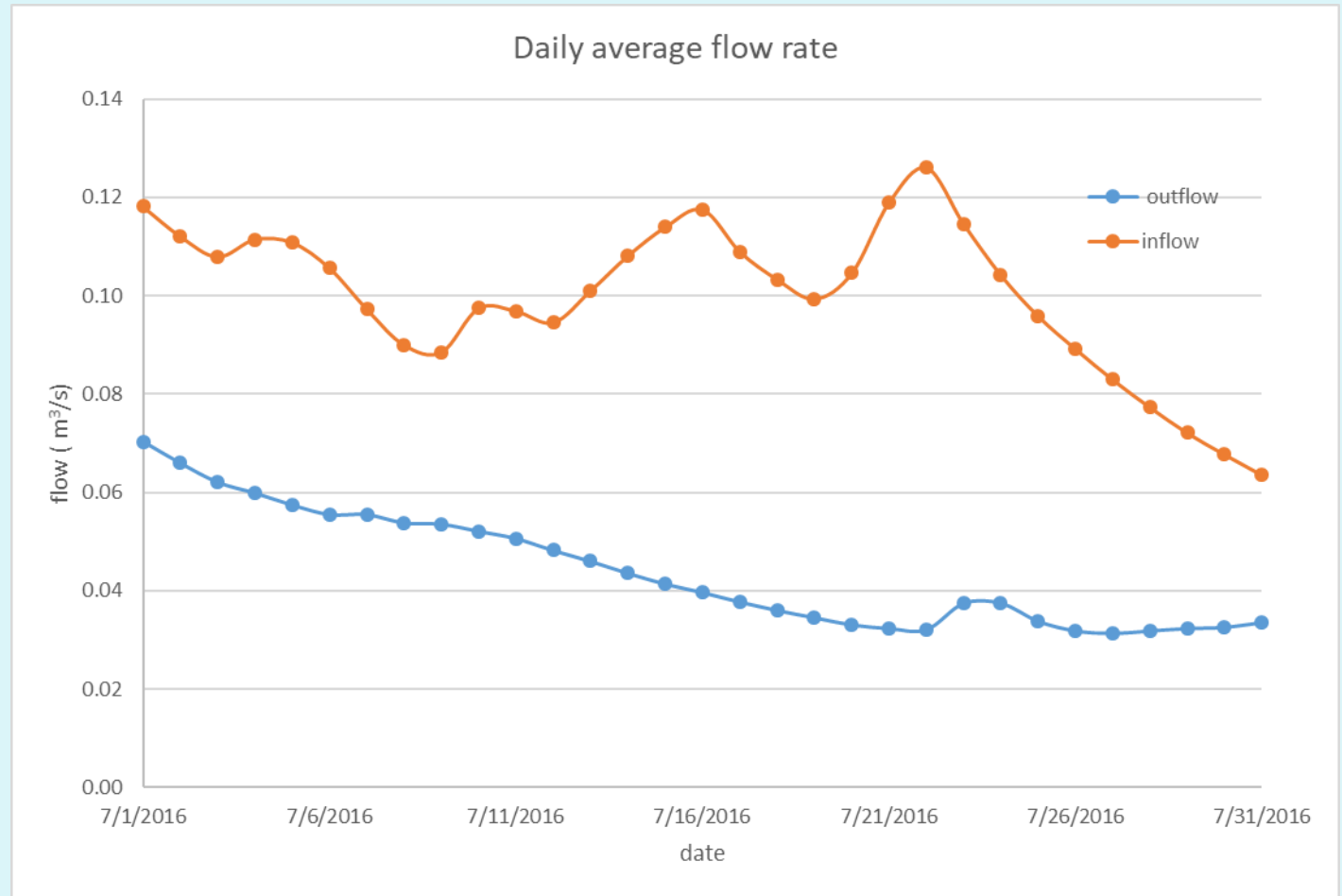
$$\Delta V = (Q_{in} - Q_{out}) \Delta t$$

# Brief aside: Data and abstractions

$Q_{in}$  and  $Q_{out}$  data from the TMF are here given as daily averages.

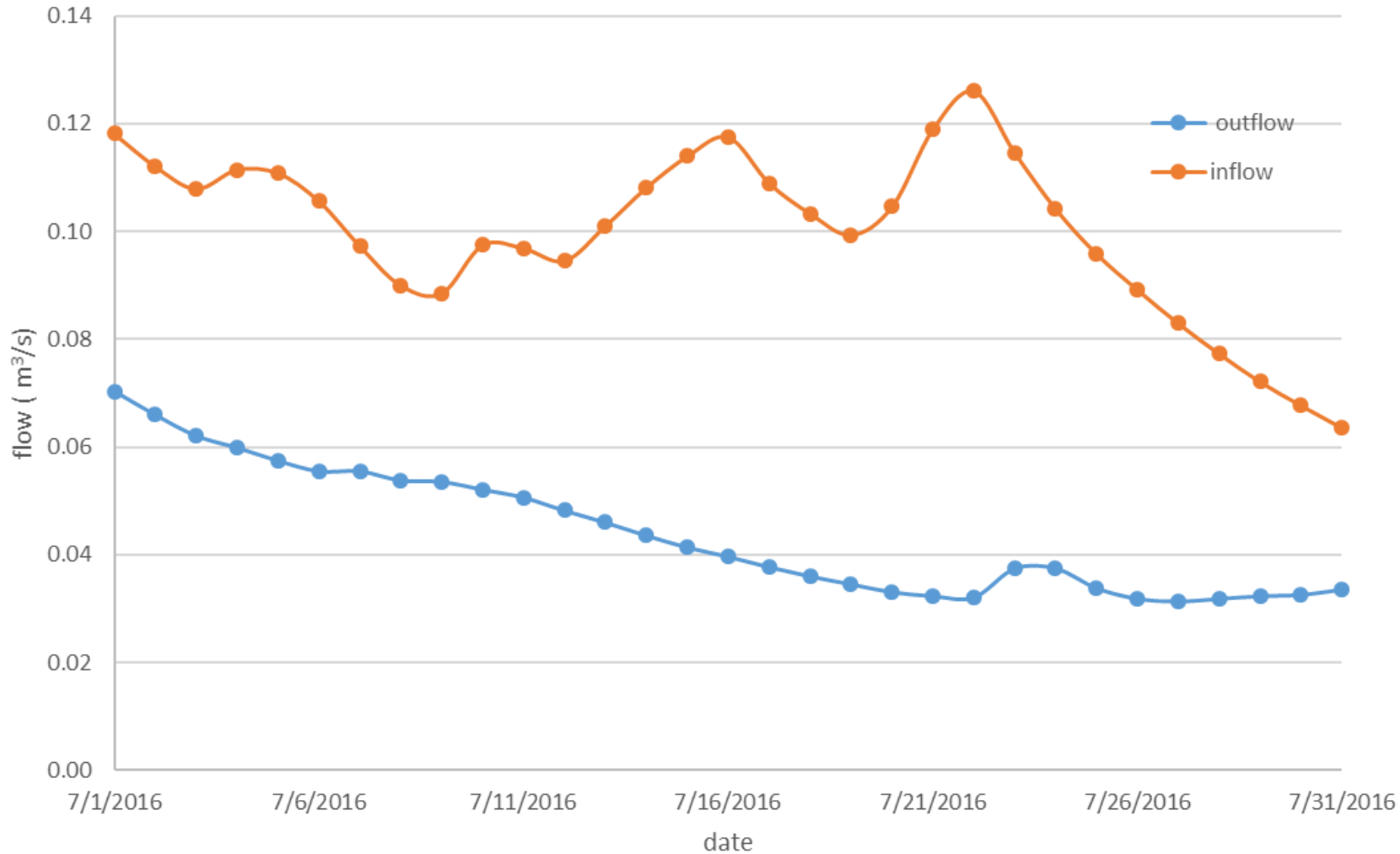
In reality,  $Q_{in}$  and  $Q_{out}$  vary in time, and can be conceptualized as *functions* of time, indicated as  $Q_{in}(t)$  and  $Q_{out}(t)$

To minimize clutter, they are often written just as  $Q_{in}$ , and  $Q_{out}$ .



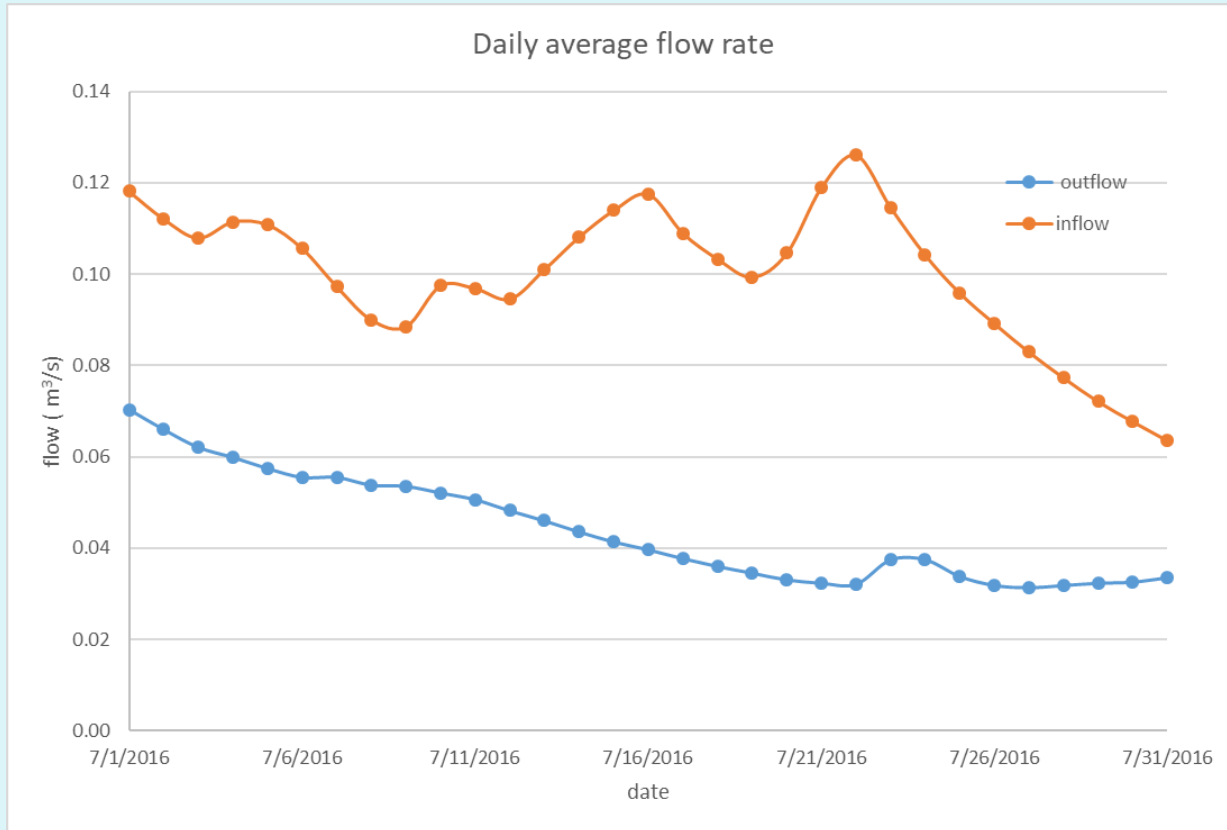
# Daily average flow

Daily average flow rate



	flow in $\text{m}^3/2$	
	outflow	inflow
2016-07-01	0.0703	0.1181
2016-07-02	0.0660	0.1121
2016-07-03	0.0621	0.1079
2016-07-04	0.0599	0.1115
2016-07-05	0.0574	0.1108
2016-07-06	0.0555	0.1056
2016-07-07	0.0555	0.0973
2016-07-08	0.0538	0.0900
2016-07-09	0.0535	0.0885
2016-07-10	0.0521	0.0975
2016-07-11	0.0506	0.0969
2016-07-12	0.0482	0.0946
2016-07-13	0.0460	0.1010
2016-07-14	0.0436	0.1081
2016-07-15	0.0414	0.1140
2016-07-16	0.0397	0.1175
2016-07-17	0.0377	0.1090
2016-07-18	0.0360	0.1033
2016-07-19	0.0346	0.0994
2016-07-20	0.0331	0.1046
2016-07-21	0.0324	0.1190
2016-07-22	0.0321	0.1260
2016-07-23	0.0375	0.1146
2016-07-24	0.0375	0.1042
2016-07-25	0.0338	0.0958
2016-07-26	0.0319	0.0892
2016-07-27	0.0314	0.0829
2016-07-28	0.0319	0.0773
2016-07-29	0.0324	0.0721
2016-07-30	0.0326	0.0677
2016-07-31	0.0336	0.0635

# Compute daily mass



Describe an algorithm to compute the mass of water in the pond at the end of each day.



# Intro to differential equations

# Differential equation

The mass model  $\Delta m = m_{in} - m_{out}$  can be re-expressed as:

$$\Delta V = (Q_{in} - Q_{out})\Delta t$$

Rearrange

$$\frac{\Delta V}{\Delta t} = (Q_{in} - Q_{out})$$

Consider smaller time intervals  $\Delta t \rightarrow 0$

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt} = (Q_{in} - Q_{out})$$

Or

$$\frac{dV}{dt} = (Q_{in} - Q_{out})$$

# Differential equation

A differential equation is an equation involving an unknown function  $y = f(x)$  and one or more of its derivatives. A solution to a differential equation is a function  $y = f(x)$  that satisfies the differential equation when  $f$  and its derivatives are substituted into the equation.

[See here](#)

In our case, the unknown function  $f$  is  $V$ , the volume of water in the TMF, and the independent variable is time  $t$ .

$$\frac{dV}{dt} = (Q_{in} - Q_{out})$$



# Differential equation

What do we gain going from the discrete

$$\Delta V = (Q_{in} - Q_{out})\Delta t$$

to the differential equation?

$$\frac{dV}{dt} = (Q_{in} - Q_{out})$$

# Solving differential equations with calculus

**Analytic solutions** – can be written in a “closed form” in terms of known functions.

**Numerical solutions** – typically discrete approximations; no explicit functions.

**Differential equations courses** - typically cover methods to derive analytical solutions



# Simple example

If  $(Q_{in} - Q_{out}) = 0.05 \frac{m^3}{s}$  (constant, not changing in time), then

$$\frac{dV}{dt} = 0.05$$

This is an example of a separable ordinary differential equation :

$$dV = 0.05 dt$$

Integrate both sides

$$\int dV = \int 0.05 dt$$

Answer

$$V(t) = 0.05 t + const$$



# Simple example

$$V(t) = 0.05 t + \text{const}$$

Does the solution satisfy the differential equation?

$$\frac{dV(t)}{dt} = \frac{d}{dt} (0.05t + \text{const}) = 0.05?$$

$$\frac{d}{dt} (0.05t + \text{const}) = \frac{d(0.05t)}{dt} + \frac{d(\text{const})}{dt} = 0.05 \quad \checkmark$$

The solution  $V(t) = 0.05 t + \text{const}$  is called a general solution.

To find the particular solution to our problem, need to determine the value of *const*.



# Simple example

$$V(t) = 0.05 t + \textit{const}$$

Find *const* from the *initial condition*. This is information about the system that must be known to solve the problem. In this case, we must know the volume of water in the TMF at  $t=0$ ,  $V(0)$ , to know the volume of water at later times.

If at time  $t = 0$ ,  $V(0) = 8.1 \times 10^6 m^3$ , then

$$\begin{aligned} V(0) = 8.1 \times 10^6 m^3 &= 0.05 (0) + \textit{const} \\ \Rightarrow \textit{const} &= 8.1 \times 10^6 m^3 \end{aligned}$$

So the volume of water in the TMF through time is given by:

$$V(t) = 0.05 t + 8.1 \times 10^6 \text{ for } t \text{ in units of seconds, and } V \text{ } m^3$$



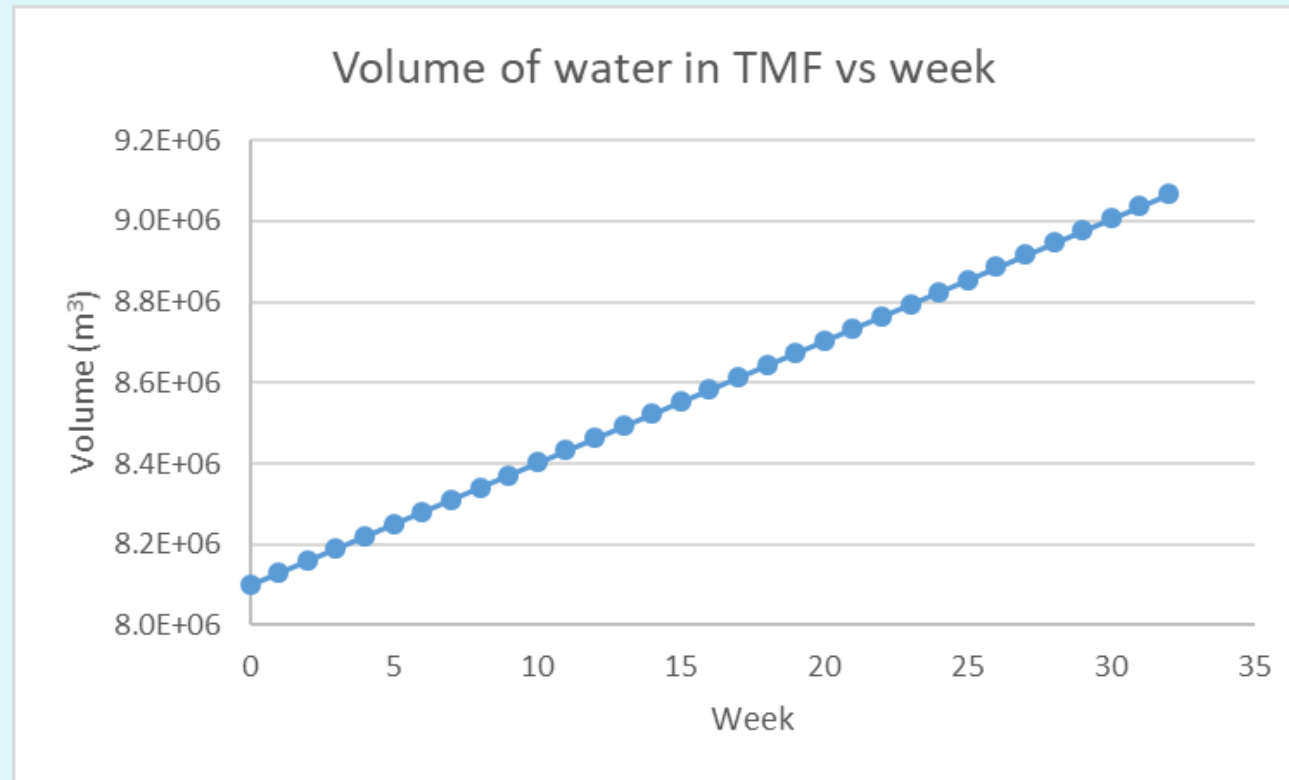


# Simple example

The particular solution is:

$$V(t) = 0.05 t + 8.1 \times 10^6$$

for  $t$  in units of seconds, and  $V \text{ m}^3$



# Conservation law

Water balance is a **Conservation law**

**Over time period  $\Delta t$ :**

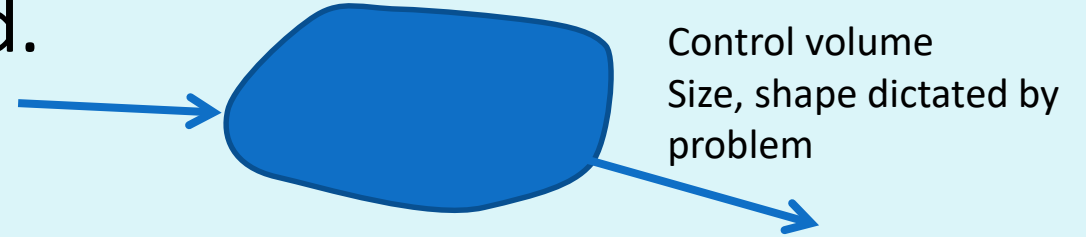
**Change = inflows – outflows**

**Conservation of mass of water in tailings management facility**



# General conservation law

Define a ***control volume*** - a volume, fixed in space and time where conservation is considered.



For a conserved property (typically mass, energy, momentum):

$$\text{Change in quantity of property in control volume over time period } \Delta t = \text{net flux of property across the boundary of the control in time period } \Delta t + \text{changes within the volume over } \Delta t.$$

What was the control volume in the TMF example?

# Conservation law



$$\frac{dV}{dt} = (Q_{in} - Q_{out})$$

Change in quantity of property in control volume over time period  $\Delta t$

=

net flux of property across the boundary of the control in time period  $\Delta t$  + changes within the volume over  $\Delta t$ .