

**Instructions**

- Read the examination before beginning.
- Answer questions on the paper, and if provided, write the final answer in the designated space at the end of the question.
- Be as precise and clear as possible – you will lose points if your answer is correct but not comprehensible.
- Point form answers are acceptable.
- If you get stuck, make an assumption, state what it is and try to carry on.
- This is a closed book examination.
- Calculators are allowed.
- You have 30 minutes for the examination. No extensions will be permitted.
- This exam consists of **2 pages**. The last page contains formulas and tables. GOOD LUCK!

**15 (+2 bonus) points total.**

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- 1) **1 point. True or False** (circle your response): *This sentence is false.*
- 2) **1 point.** Show by analyzing the dimensions that the following expression for the area of a circle is wrong:  $A = \pi d/2$  where  $A$  is area of the circle and  $d$  is the diameter of the circle.

$$A [L^2] = \frac{\pi[\cdot]d[L]}{2}$$

Just need to show that the dimensions of left and right side are different. No points if they don't get one of the dimensions right.

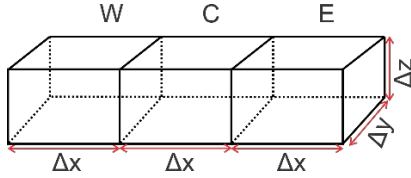
- 3) **6 points.** Write a python function to evaluates the function

$$f(x, y) = ax^2 + by^3 + c$$

Your function should

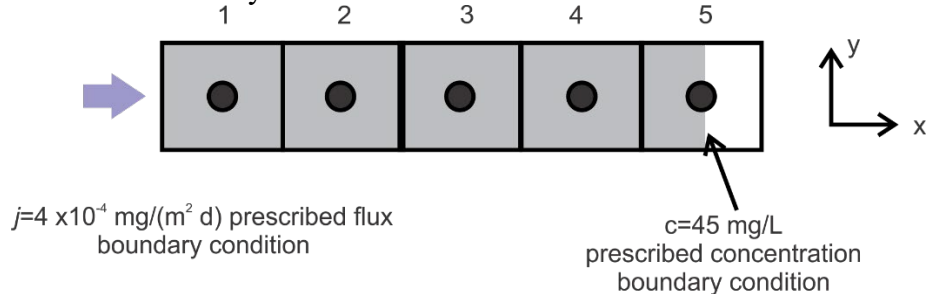
- a) Set default values of  $a = 0$ ,  $b = 0$ ,  $c = 0$
- b) Require both  $x$  and  $y$
- c) Work for  $x$  and  $y$  as scalars, python lists, or numpy arrays

- 4) **4 points.** Write the general finite-volume W-C-E stencil for 1-dimensional, steady-state diffusion in porous media, without sources or sinks, in terms of the concentrations  $c_C, c_W, c_E$ , the gridblock dimensions  $\Delta x, \Delta y, \Delta z$ , the diffusion coefficient  $D \left[ \frac{L^2}{T} \right]$ , and porosity  $\theta$ , and where Fick's law in porous media is given as  $j_x = -D\theta \frac{\partial c}{\partial x}, j_y = -D\theta \frac{\partial c}{\partial y}, j_z = -D\theta \frac{\partial c}{\partial z}$ .



$$\left( D\theta \frac{c_W - c_C}{\Delta x} + D\theta \frac{c_E - c_C}{\Delta x} \right) \Delta y \Delta z = 0$$

- 5) Consider the following 1-D finite-volume grid with 5 gridblocks numbered as shown for a 1-D diffusion problem with boundary conditions as shown:



- a) **4 points.** What is the **net mass that enters or leaves gridblock 3 over a time interval  $\Delta t = 2.5$  h**, if the concentrations in gridblocks 2, 3 and 4 over that time interval are  $c_2 = 63.5 \frac{mg}{L}, c_3 = 54.8 \frac{mg}{L}, c_4 = 50.1 \frac{mg}{L}$ , where  $\Delta x = 2.0$  m,  $\Delta y = 1.5$  m,  $\Delta z = 1$  m, the diffusion coefficient is  $D = 7 \times 10^{-9} m^2/s$ , the porosity is  $\theta = 0.3$ .

Net flux **rate** just  $J_{23} + J_{43} = \left( D\theta \frac{c_W - c_C}{\Delta x} + D\theta \frac{c_E - c_C}{\Delta x} \right) \Delta y \Delta z$  so the net mass over  $\Delta t = 2.5$  hrs is

$$\left( D\theta \frac{c_W - c_C}{\Delta x} + D\theta \frac{c_E - c_C}{\Delta x} \right) \Delta y \Delta z \Delta t$$

$$\text{Mass from 2 to 3: } D\theta \frac{c_W - c_C}{\Delta x} \Delta y \Delta z \Delta t = 9.25e-5 \text{ mg}$$

$$\text{Mass from 4 to 3} = -5.0e-5 \text{ mg}$$

$$\text{Net mass into 3: } 4.25e-5 \text{ mg}$$

Dth	x	y	z	j bc	Dth y z/x
2.1E-09	2	1.5	1	4.63E-09	1.575E-09
Question	j			mass	
from 2 to 3	6.8513E-09	flux 2 to 3	9.24919E-05		
from 4 to 3	-3.701E-09	flux 4 to 3	-4.99669E-05		
net	3.15E-09	- net rate flux into gridblock 3 in mg/(m <sup>2</sup> s)			
mass over 2.5 h	4.2525E-05	answer in (mg) over 2.5 h		4.2525E-05	

b) **2 points.** Is the system in steady state? Explain. **Not at steady state because the mass in the gridblock is changing over time.**